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EVALUATION OF FULL SEISMIC MOMENT TENSOR FROM ISOTROPIC, CLVD AND DOUBLE-COUPLE COMPONENTS

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ABSTRACT
Formulas for full seismic moment tensor composition are present, i.e. moment tensor is express as a function of ISO, CLVD, DC, strike, dip, rake, where ISO is amount of isotropic part, CLVD is amount of compensated liner-vector dipole and DC is amount of pure double couple. Two forms of final formulas are given: *i,* two matrixes multiplication, *ii*, extension of "classical" formulas for 6 independent moment tensor elements.

KEYWORDS: evaluation of full seismic moment tensor, point source representation

MOTIVATION

Contemporary studies of seismic sources commonly invert for full seismic moment tensor (hereafter MT) – i.e. they do not look only for (pure) double couple (DC) but decompose MT also into its isotropic (ISO) and compensated linear-vector dipole (CLVD) parts. To be able to perform synthetic computation as well as to have testing data for programs for real data processing, we derived formulas for full MT composition, i.e., express seismic MT in form of

$$m_{ij} = m_{ij}(\mathrm{ISO},\mathrm{CLVD},\mathrm{DC}, \text{strike, dip, rake}), \qquad (1)$$

where m_{ij} are elements of moment tensor **M**. Note, that such formula is an extension of formula (1) from box 4.4 in Aki and Richards (1980), where MT is consider to be pure DC dipole, i.e. it is expressed as a function (only) of strike, dip and rake.

DEFINITIONS

In the paper we suppose point source representation and MT decomposition into ISO, CLVD and DC parts.[1]

Description of a seismic source by MT is a standard approach, however its decomposition has not yet been fully unified and different definitions are used - see e.g. Šílený and Pšenčík (1995) or Dahm (1996). As the most consistent seems to be the approach suggested recently by Vavryčuk (2001), formulas (7) and (8):

$$\mathrm{ISO} = \mathrm{trace}(\mathbf{M}) / M_{|max}|/ 3, \qquad (2a)$$

$$\mathrm{CLVD} = - 2M^*_{|min|} / | M^*_{|max|} |(1 - |\mathrm{ISO}|), \qquad (2b)$$

$$\mathrm{DC}=1 - |\mathrm{ISO}| - |\mathrm{CLVD}| , \qquad (2c)$$

where $M_{|max|}$ denotes that eigenvalue of **M**, which has the maximum absolute value (and analogously for $M_{|min|}$); symbol M^* denotes deviatoric part of **M**; absolute values of ISO, CLVD and DC range in interval <0, 1>. This definition of decomposition has following basic features: the value of DC is always positive, and the ISO and CLVD are positive for tensile source, but negative for compressive source. The sum of their absolute values is always a unit.

We shell also define vectors **t** – tension, **p** – pressure and their combination **b** in a standard way as:

$$\mathbf{t} = (\mathbf{v}+\mathbf{u})\big/\sqrt{2}\,,$$
$$\mathbf{b} = \mathbf{v} \times \mathbf{u}, \qquad (3)$$
$$\mathbf{p} = (\mathbf{v} - \mathbf{u})\big/\sqrt{2}\,,$$

where **u** is a slip vector and **v** is fault normal expressed in terms of strike, dip and rake – see definition in Fig. 4.20 and formulas 4.83 in Aki and Richards (1980).

[1] Even if MT can be decomposed in several different manners (e.g. 2DC + ISO, etc. – see Jost and Herrmann, 1989 or Julian et al., 1998), the decomposition into ISO, CLVD and DC seems to be used most frequently.

MT COMPOSITION

Following Jost and Herrmann (1989), formula (23) for isotropic part of MT characterized by parameter V and formula (35) for its DC - CLVD decomposition characterized by parameter F, we can write their sum

$$\mathbf{M^{eig}} = \begin{bmatrix} -F+V & 0 & 0 \\ 0 & F-1+V & 0 \\ 0 & 0 & 1+V \end{bmatrix}, \tag{4}$$

where $\mathbf{M^{eig}}$ is MT oriented in direction of eigenvectors **t**, **b** and **p** (Jost and Herrmann, 1989).

If we take $M_{|max|} = 1 + V$, which is justify at least for ISO > 0 with consideration that parameter F ranges in interval <0, 0.5> (Jost and Herrmann, 1989), we can put (4) into (2a) and express volumetric parameter V as

$$V = ISO/(1- ISO) \ , \tag{5}$$

Then, if we take $M^*_{|min|} = F\text{-}1$ and $M^*_{|max|} = 1$, supposing here that ISO > 0 and CLVD > 0, we can put (4) and (5) into (2b) and express deviatoric parameter F as

$$F = 1+(-0.5\ CLVD)/(1- ISO)\ . \tag{6}$$

Generally, values of ISO and CLVD can be positive as well as negative (see above). However, to avoid ambiguity with analytical expression of their absolute values as well as with sorting of their values, we have to restrict the derivation only for their positive values. This restriction holds also for all the following formulas, which contain expression for parameter V or F respectively.

We rotate $\mathbf{M^{eig}}$ to obtain moment tensor **M** in coordinate system corresponding to the coordinates in which strike, dip and rake are measured, i.e. right-hand oriented system North, East, Z(down) - in accord with definition of Aki and Richards (1980), Fig. 4.20:

$$\mathbf{M} = 1/n\ \mathbf{P}^T\ \mathbf{M^{eig}}\ \mathbf{P}\ , \tag{7}$$

where $\mathbf{P}^T$ is transposed rotation matrix **P** given

$$\mathbf{P} = \begin{bmatrix} \mathbf{b} \\ \mathbf{p} \\ \mathbf{t} \end{bmatrix}, \tag{8}$$

and n is a normalization factor[2] to assure unit size of MT given

$$n = \sqrt{\left((\text{-}F+V)^2 + (F\text{-}1+V)^2 + (1+V)^2\right)/2}\ . \tag{9}$$

To obtain an explicit formulas for 6 independent MT elements m_{ij}, we evaluate expression (7) with using (3) and putting in the definition for vectors **v** and **u** as a function of strike, dip and rake – see formulas 4.83 in Aki and Richards (1980).

$$\begin{aligned} m_{11} &= M_0/n \\ &\{(\text{-}F+V)\cdot\left(\cos(\delta)\left(\cos(\lambda)\sin(\Phi_S)\text{-}\cos(\delta)\sin(\lambda)\cos(\Phi_S)\right)\text{-}\sin^2(\delta)\cos(\Phi_S)\sin(\lambda)\right)^2 \\ &+(F\text{-}1+V)/2\cdot\left(\text{-}\sin(\delta)\sin(\Phi_S)\text{-}\cos(\lambda)\cos(\Phi_S)\text{-}\cos(\delta)\sin(\lambda)\sin(\Phi_S)\right)^2 \\ &+(1+V)/2\cdot\left(\text{-}\sin(\delta)\sin(\Phi_S)+\cos(\lambda)\cos(\Phi_S)+\cos(\delta)\sin(\lambda)\sin(\Phi_S)\right)^2\} \end{aligned} \tag{10}$$

$$\begin{aligned} m_{12} &= M_0/n \\ &\{3F/2\cdot\left(\cos^2(\delta)\cos^2(\lambda)\sin(\Phi_S)\cos(\Phi_S)+\cos(\delta)\cos(\lambda)\sin(\lambda)+\cos^2(\lambda)\cos(\Phi_S)\sin(\Phi_S)\text{-}\sin(\Phi_S)\cos(\varphi_S)\right) \\ &\text{-}F\cdot\left(\sin(\delta)\sin(\Phi_S)\cos(\delta)\sin(\lambda)\cos(\Phi_S)+\cos(\lambda)\cos^2(\Phi_S)\sin(\delta)+3\cos(\lambda)\cos^2(\Phi_S)\cos(\delta)\sin(\lambda)\right) \\ &+ F/2\cdot\sin(\delta)\cos(\lambda)+2\cos(\lambda)\cos^2(\Phi_S)\sin(\delta)\text{-}\sin(\delta)\cos(\lambda)+2\sin(\delta)\sin(\Phi_S)\cos(\delta)\sin(\lambda)\cos(\Phi_S)\} \end{aligned}$$

$$\begin{aligned} m_{13} &= M_0/n \\ &\{3F/2\cdot\left(\cos(\delta)\cos^2(\lambda)\sin(\Phi_S)\sin(\delta)\text{-}\cos(\lambda)\cos(\Phi_S)\sin(\lambda)\sin(\delta)\right)+F\cdot\cos^2(\delta)\sin(\lambda)\sin(\Phi_S) \\ &+F/2\cdot\left(\cos(\lambda)\cos(\Phi_S)\cos(\delta)\text{-}\sin(\Phi_S)\sin(\lambda)\right)+\sin(\lambda)\sin(\Phi_S)\text{-}\cos(\delta)\cos(\lambda)\cos(\Phi_S)\text{-}2\cos^2(\delta)\sin(\lambda)\sin(\Phi_S)\} \end{aligned}$$

[2] This is valid if norm of **M** is calculated as L_2 norm. N.B. that there are possible other definitions of tensor or matrix norms - c.f. e.g. MATLAB function NORM for matrix.

$$m_{22} = M_0/n$$
$$\{(-F+V)\cdot\left(-\cos(\delta)(\cos(\lambda)\cos(\Phi_S)+\cos(\delta)\sin(\lambda)\sin(\Phi_S))-\sin(\lambda)\sin^2(\delta)\sin(\Phi_S)\right)^2$$
$$+(F-1+V)/2\cdot\left(\sin(\delta)\cos(\Phi_S)-\cos(\lambda)\sin(\Phi_S)+\cos(\delta)\sin(\lambda)\cos(\Phi_S)\right)^2$$
$$+(1+V)/2\cdot\left(\sin(\delta)\cos(\Phi_S)+\cos(\lambda)\sin(\Phi_S)-\cos(\delta)\sin(\lambda)\cos(\Phi_S)\right)^2\}$$

$$m_{23} = M_0/n$$
$$\{-3F/2\ \cdot\left(\cos(\lambda)\sin(\Phi_S)\sin(\lambda)\sin(\delta)+\cos(\delta)\cos^2(\lambda)\cos(\Phi_S)\sin(\delta)\right)$$
$$-F\cdot\cos^2(\delta)\sin(\lambda)\cos(\Phi_S)+F/2\ \cdot\left(\cos(\lambda)\sin(\Phi_S)\cos(\delta)+\sin(\lambda)\cos(\Phi_S)\right)$$
$$+2\cos^2(\delta)\sin(\lambda)\cos(\Phi_S)-\cos(\lambda)\sin(\Phi_S)\cos(\delta)-\sin(\lambda)\cos(\Phi_S)\}$$

$$m_{33} = M_0/n$$
$$\{V+F/2-3F/2\cdot\cos^2(\lambda)\sin^2(\delta)-F\cdot\cos(\delta)\sin(\lambda)\sin(\delta)+2\cos(\delta)\sin(\lambda)\sin(\delta)\},$$

where M_0 is a scalar seismic moment, Φ_S stands for strike, δ for dip and λ for rake; indexes 1,2,3 are equivalent to coordinate system NEZ. Remember again, that the formulas were derived under the assumption of ISO > 0 and CLVD > 0. Formulas (7) and (10) are not restricted on MT decomposition given by (2), but can be used for any other definition of CLVD-DC decomposition, if expression (5) and (6) for parameters V and F are properly modified.

CONCLUSION

We present formulas for evaluation of full seismic tensor moment as a function of ISO, CLVD, DC, strike, dip, rake in two forms: *i,* in form of multiplication of two matrixes (7) and *ii,* in individual expression for 6 independent MT elements (10). From practical point of view, the first form is more convenient for programming,[3] the latest form is an extension of "classical" formulas for pure double couple seismic source. The derived formulas can be useful for numerical simulations as well as for generation of testing data.[4]

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[3] We programmed presented formulas (the matrix version) and they are available under a request in form of MATLAB functions. There is a function for composition of full MT (in form of M=M(ISO, CLVD, strike, dip, rake) and an inverse function for MT decomposing (in form of [ISO, CLVD, M_0, strike1, dip1, rake2, strike2, dip2, rake2]=M).

[4] Derivation of the formulas is not too complicated in principal, however it takes a while to perform it and the formulas can therefore be exploited e.g. in tests of MT decomposition. Of course, in real data processing MT decomposition would be required definitely more often than MT composition.